**UNIVERSITY OF MISSISSIPPI**
Department of Physics and Astronomy
Electromagnetism II (Phys. 402) — Prof. Leo C. Stein — Spring 2020

**Problem Set 6**

**Due**: Wednesday, Apr. 1, 2020, by 5PM

As with research, feel free to collaborate and get help from each other! But the solutions you hand in must be your own work. All book problem numbers refer to the third edition of Griffiths, unless otherwise noted. I know we don't all have the same edition, so I also briefly describe the topic of the problem.

1. **Energy/momentum transfer**. Suppose a plane electromagnetic wave is traveling through vacuum (constants $\epsilon_0$ and $\mu_0$) in the $\hat{z}$ direction, with frequency $\omega$, with the electric field linearly polarized in the $\hat{x}$ direction with amplitude $E_{0I}$.

   (a) This wave is incident on a perfectly absorbing sheet lying in the $x - y$ plane. How much energy does the sheet absorb per unit time, per unit area? How much momentum?

   (b) Now suppose we replace the perfect absorber with a perfectly reflecting mirror. How much energy does the mirror absorb per unit time, per unit area? How much momentum?

   Next suppose we replace the perfect reflector with a partially-transmitting sheet of linear medium with electric permittivity $\epsilon_2$ and magnetic permeability $\mu_0$. This sheet has some finite thickness but let us focus only on the first interface, between vacuum and the material, and ignore everything that happens downstream.

   (c) For normal incidence, what are the reflected and transmitted electric and magnetic fields in terms of the incident field?

   (d) What is the momentum density $\wp$ (which is real, not complex) in the transmitted field? (Hint: how do the permittivity and permeability enter into $\wp$?)

   (e) What is the momentum density, separately, in (i) the incident field, and (ii) the reflected field?

   (f) What is the momentum density in the *sum* of the incident and reflected fields?

   (g) Which of these results do you think is the correct way to compute the momentum transferred to the partially-transmitting sheet? Justify your claim.

2. **Reflection with horizontal polarization** (long). In lecture we went through the derivation of reflection where the electric field is "vertically" polarized, i.e. with the $\boldsymbol{E}$ field lying in the $x - z$ plane of incidence. Redo the calculation but with the horizontal polarization, i.e. with $\boldsymbol{E} \propto \hat{y}$.

   (a) Write down the four boundary conditions, evaluated with the appropriate parallel/perpendicular electric and magnetic fields, in terms of the angles $\theta_I, \theta_T$ etc.

   (b) Solve the resulting linear system for the ratios $\tilde{\boldsymbol{E}}_{0R}/\tilde{\boldsymbol{E}}_{0I}$ and $\tilde{\boldsymbol{E}}_{0T}/\tilde{\boldsymbol{E}}_{0I}$, in terms of the earlier variables $\alpha \equiv \cos\theta_T / \cos\theta_I$ and $\beta \equiv \mu_1 v_1/\mu_2 v_2$.

   (c) Give an equation for Brewster's angle $\theta_B$ where there is no (horizontal) reflection. Assuming that $\mu_1 \approx \mu_2$ to simplify, what do you find for the no-reflection condition?

   (d) Check that the reflection and transmission coefficients add up to 1 (recall that the transmission coefficient is the ratio of intensities, rather than the square of the ratio of electric fields).

3. Silicon carbide has an index of refraction of $n = 2.65$.
   (a) Plot the ratios $E_{0R}/E_{0I}$ and $E_{0T}/E_{0I}$ as a function of $\theta_I$, for the interface between SiC and air (assuming $\mu_1 = \mu_2 = \mu_0$).
   (b) What are the values for the two amplitude ratios at normal incidence?
   (c) What is Brewster's angle?
   (d) What is the "crossover" angle, where the reflection and transmission amplitudes are equal?
4. Griffiths 9.22a-b (phase and group velocities in deep water waves, and quantum mechanics). Note that Griffiths writes "wave velocity" for what everyone calls the *phase* velocity.