UNIVERSITY OF MISSISSIPPI
Department of Physics and Astronomy
Electromagnetism II (Phys. 402) — Prof. Leo C. Stein — Spring 2020

Problem Set 3

Due: Monday, Feb. 24, 2019, by 5PM

As with research, feel free to collaborate and get help from each other! But the solutions you hand in must be your own work. All book problem numbers refer to the third edition of Griffiths, unless otherwise noted. I know we don't all have the same edition, so I also briefly describe the topic of the problem.

1. Griffiths problem 7.59 (proving Alfven's theorem).

2. **A highly conducting, magnetized plasma – part 2**. Last week, we saw that in a highly conducting plasma, in the limit of $\sigma \to \infty$, we would find the conditions:

   $$\boldsymbol{E} = -\boldsymbol{v} \times \boldsymbol{B}\,, \tag{1}$$

   therefore

   $$\boldsymbol{E} \cdot \boldsymbol{B} = 0\,. \tag{2}$$

   Then we decomposed $\boldsymbol{v} = v_\parallel \hat{\boldsymbol{B}} + \boldsymbol{v}_\perp$, and found

   $$\boldsymbol{v}_\perp = \boldsymbol{E} \times \boldsymbol{B}/B^2\,. \tag{3}$$

   (a) Starting from Eq. (2), take the time derivative of this result. Plug in for the time derivatives of the electromagnetic fields using Maxwell's equations. You should now be able to solve for $\boldsymbol{v} \cdot \boldsymbol{B}$, allowing you to find the parallel component $v_\parallel$.

   (b) Finally, sum up by rewriting $\boldsymbol{J}$ purely in terms of *only* the $\boldsymbol{E}$ and $\boldsymbol{B}$ fields and their derivatives.

3. Griffiths problem 8.5a-d (Infinite parallel-plate capacitor's stress tensor, force per unit area, momentum flux, recoil per unit area).

4. Griffiths problem 8.9a-b (Solenoid with a ring outside, energy flux).

5. **Ramping the current in a solenoid**. Suppose we have a solenoid of radius a aligned with the $\hat{z}$ axis with n turns per unit length.

   (a) Suppose we turn on the current through the solenoid so that between $t = 0$ and $t = \tau$, the current increases linearly, $I(t) = I_1 \frac{t}{\tau}$. What is the magnetic field $\boldsymbol{B}$ in the solenoid? From Maxwell's equations, what is the electric field $\boldsymbol{E}$?

   (b) Find the energy flux and the momentum density in the electromagnetic field inside the solenoid.

   (c) Find the Maxwell stress tensor in the basis of $\hat{z}, \hat{s}, \hat{\phi}$ (i.e. you are looking for components like $T_{\phi\phi}$, T_{zs}, etc.).