UNIVERSITY OF MISSISSIPPI
Department of Physics and Astronomy
Electromagnetism II (Phys. 402) — Prof. Leo C. Stein — Spring 2019

Problem Set 6

Due: Wednesday, Apr. 3, 2019, by 5PM

As with research, feel free to collaborate and get help from each other! But the solutions you hand in must be your own work. All book problem numbers refer to the third edition of Griffiths, unless otherwise noted. I know we don't all have the same edition, so I also briefly describe the topic of the problem.

1. **Reflection with horizontal polarization** (long). In lecture we went through the derivation of reflection where the electric field is "vertically" polarized, i.e. with the $\boldsymbol{E}$ field lying in the $x - z$ plane of incidence. Redo the calculation but with the horizontal polarization, i.e. with $\boldsymbol{E} \propto \hat{y}$.
   (a) Write down the four boundary conditions, evaluated with the appropriate parallel/perpendicular electric and magnetic fields, in terms of the angles θ_I, θ_T etc.
   (b) Solve the resulting linear system for the ratios $\tilde{\boldsymbol{E}}_{0R}/\tilde{\boldsymbol{E}}_{0I}$ and $\tilde{\boldsymbol{E}}_{0T}/\tilde{\boldsymbol{E}}_{0I}$, in terms of the earlier variables $\alpha \equiv \cos\theta_T/\cos\theta_I$ and $\beta \equiv \mu_1 v_1/\mu_2 v_2$.
   (c) Give and equation for Brewster's angle θ_B where there is no (horizontal) reflection. Assuming that $\mu_1 \approx \mu_2$ to simplify, what do you find for the no-reflection condition?
   (d) Check that the reflection and transmission coefficients add up to 1 (recall that the transmission coefficient is the ratio of intensities, rather than the square of the ratio of electric fields).

2. Silicon carbide has an index of refraction of $n = 2.65$.
   (a) Plot the ratios E_{0R}/E_{0I} and E_{0T}/E_{0I} as a function of θ_I, for the interface between SiC and air (assuming $\mu_1 = \mu_2 = \mu_0$).
   (b) What are the values for the two amplitude ratios at normal incidence?
   (c) What is Brewster's angle?
   (d) What is the "crossover" angle, where the reflection and transmission amplitudes are equal?

3. (a) Suppose you embedded some free charge in a piece of glass. About how long would it take for the charge to flow to the surface?
   (b) Silver is an excellent conductor, but it's expensive. Suppose you were designing a microwave experiment to operate at a frequency of 10^{10}Hz. How thick would you make the silver coatings?
   (c) Find the wavelength and propagation speed in copper for radio waves at 1 MHz. Compare the corresponding values in air (or vacuum).

4. Griffiths 9.19a-c (asymptotic limits of skin depth for very good/bad conductors; and magnetic field lag and strength in a good conductor). For part c, in SI units, the ratio of B/E is not dimensionless, so it's silly to look at the numerical value by itself. Rather, find the ratio of B/E in a good conductor, and compare it to the ratio in vacuum.

5. Griffiths 9.22a-b (phase and group velocities in deep water waves, and quantum mechanics). Note that Griffiths writes “wave velocity” for what everyone calls the *phase* velocity.

6. Consider a rectangular waveguide with sides 3.42 cm × 1.515 cm. What TE modes will propagate in this waveguide, if the driving frequency is 1.13×10^{10}Hz? Suppose you want to excite only one TE mode. What range of frequencies could you use? What are the corresponding wavelengths of those frequencies when they are in free space?

7. **Resonant cavity modes** (long). Take what you've learned in the analysis of waveguides and apply it to a resonant cavity. Suppose we have a hollow rectangular box of side lengths $a \geq b \geq d$, and this box is made out of an excellent conductor. Solve Maxwell's equations subject to the appropriate boundary conditions at all the surfaces. Find the modes that are possible, which should now be labeled by three integers (l, m, n), and find the associated frequency ω_{lmn}. What is the general solution for $\boldsymbol{E}$ and $\boldsymbol{B}$ in one of these modes?