UNIVERSITY OF MISSISSIPPI
Department of Physics and Astronomy
Electromagnetism II (Phys. 402) — Prof. Leo C. Stein — Spring 2019

Problem Set 4

Due: Friday, Mar. 8, 2019, by 5PM

As with research, feel free to collaborate and get help from each other! But the solutions you hand in must be your own work. All book problem numbers refer to the third edition of Griffiths, unless otherwise noted. I know we don't all have the same edition, so I also briefly describe the topic of the problem.

1. Griffiths problem 9.9a-b (The real $\boldsymbol{E}$ and $\boldsymbol{B}$ fields due for two example monochromatic plane waves).

2. Griffiths problem 9.12 (The Maxwell stress tensor due to a monochromatic plane wave traveling in the z direction).

3. **Stress in index notation**. Suppose you have a monochromatic plane wave where the direction and k-number are given by the vector $\boldsymbol{k}$, or k_i in index notation; and this wave is linearly polarized with polarization vector $\boldsymbol{e}$, or e_i in index notation, where $\boldsymbol{k} \cdot \boldsymbol{e} = 0 = k_i e_i$. Find the Maxwell stress tensor T_{ij} in index notation, in terms of the above quantities.

4. **Standing electromagnetic wave**. Suppose we superimpose the following two complex, monochromatic waves, both with frequency ω: one traveling in the $\hat{z}$ direction, and the electric field is polarized in $\hat{x}$ with amplitude $\tilde{E}_0$; and the second is traveling in the $-\hat{z}$ direction, with electric field polarized in $-\hat{x}$ with the same amplitude $\tilde{E}_0$.

   (a) Write down the complex $\tilde{\boldsymbol{E}}, \tilde{\boldsymbol{B}}$ fields for each of the two waves separately.

   (b) Now superimpose them and find the *real* field that arises from the superposition.

   (c) Find the part of the period-averaged energy density due just to the electric field and comment on its pattern in space.

   (d) Now examine the total period-averaged energy density $\langle u \rangle$ (due to both the $\boldsymbol{E}$ and $\boldsymbol{B}$ fields), and again comment on its pattern in space.

   (e) Finally, find the period-averaged Poynting vector, $\langle \boldsymbol{S} \rangle$.

5. **Getting no transmission at normal incidence**.

   (a) What is the factor β (appearing in reflection/transmission at normal incidence) in terms of just ϵ's and μ's of two media at an interface?

   (b) In terms of ϵ's and μ's, how do we get no transmission from medium 1 into medium 2 at normal incidence? What property of a medium could cause this?

   (c) What does no transmission do to the reflected fields, $\tilde{\boldsymbol{E}}_R$ and $\tilde{\boldsymbol{B}}_R$?

6. Griffiths problem 9.14 (Show that reflected and transmitted waves at normal incidence must have same polarization).