UNIVERSITY OF MISSISSIPPI
Department of Physics and Astronomy
Electromagnetism II (Phys. 402) — Prof. Leo C. Stein — Spring 2019

Problem Set 2

Due: Thursday, Feb. 21, 2019, by 5PM

As with research, feel free to collaborate and get help from each other! But the solutions you hand in must be your own work. All book problem numbers refer to the third edition of Griffiths, unless otherwise noted. I know we don't all have the same edition, so I also briefly describe the topic of the problem.

1. Griffiths problem 7.30 (Mutual inductance between two tiny wire loops).
2. Much of the interstellar medium (ISM) is very low number density, typically $n \approx 1 \text{atom}/\text{cm}^3$ (you can assume it is entirely Hydrogen). There are magnetic fields permeating the galaxy with strengths like $B \approx 10\mu G$ (microGauss).
   (a) What is a typical energy density of the magnetic field in the ISM?
   (b) Suppose there is *equipartition* of energy between magnetic energy density and thermal energy density (which of course depends on density and temperature). What is a typical temperature of the ISM?
3. Griffiths 4$^{\text{th}}$ edition problem 7.49 (getting $\boldsymbol{E}$ in terms of the vector potential).
4. Griffiths problem 7.53 (ratio of EMFs in a transformer).
5. **A highly conducting, magnetized plasma**. Consider a plasma with a conductivity[1] σ, charge density ρ (that varies throughout the plasma), and where at each point the particles are moving with velocity $\boldsymbol{v}$ (that also varies from place to place). Ohm's law says that the current density is

$$\boldsymbol{J} = \sigma \boldsymbol{f} = \sigma \left(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} \right) . \tag{1}$$

   (a) Suppose the conductivity σ is taken to infinity, while the current density $\boldsymbol{J} = \rho \boldsymbol{v}$ remains finite. What relationship does this imply between the electromagnetic fields?
   (b) From the previous answer, what do you know about $\boldsymbol{E} \cdot \boldsymbol{B}$?
   (c) From these two results, you should be able to determine $\boldsymbol{v}_\perp$, the part of the plasma velocity that is perpendicular to the magnetic field [Hint: decompose $\boldsymbol{v}$ into pieces that are parallel and perpendicular to $\boldsymbol{B}$; then you might want to cross product the result of part 5a with something else].
   (d) Take the time derivative of the result of part 5b. Plug in for the time derivatives of the electromagnetic fields using Maxwell's equations. You should now be able to solve for $\boldsymbol{v} \cdot \boldsymbol{B}$, allowing you to find the parallel component $v_\parallel$.

   If you were successful in all these steps, then (in this highly conductivity regime) you can eliminate the matter sources from Maxwell's equations. This regime is relevant in many astrophysical plasmas that are very low density.

6. Griffiths problem 7.59 (proving Alfven's theorem).

[1] We will not refer to the resistivity, $1/\sigma$, which is sometimes denoted ρ. Instead we reserve the symbol ρ for the charge density.