UNIVERSITY OF MISSISSIPPI
Department of Physics and Astronomy
Electromagnetism II (Phys. 402) — Prof. Leo C. Stein — Spring 2019

Problem Set 1

Due: Thursday, Feb. 7, 2019, by 5PM

As with research, feel free to collaborate and get help from each other! But the solutions you hand in must be your own work.

1. **Practice with index notation**. Remember that we're using the *Einstein summation convention*, which means that when an index is repeated, there is an implicit sum over all the values it takes. For example, if we have two vectors $\boldsymbol{A}$ and $\boldsymbol{B}$, they each have three components A_i where $i = 1, 2, 3$ which are usually called $A_1 = A_x$, $A_2 = A_y$, and so on; then their dot product can be written as

$$\boldsymbol{A} \cdot \boldsymbol{B} = A_i B^i = \sum_{i=1}^{3} A_i B^i = A_1 B^1 + A_2 B^2 + A_3 B^3 \,. \tag{1}$$

The basic objects we have to work with are the Kronecker delta, δ_{ij}, which is 1 when $i = j$ and 0 otherwise; the Levi-Civita tensor or alternating or completely antisymmetric tensor ϵ_{ijk} which is $+1$ when $ijk = 1\,2\,3$ or a cyclic permutation ($2\,3\,1$ or $3\,1\,2$), and is -1 when $ijk = 3\,2\,1$ or a cyclic permutation, and is 0 otherwise; and ∇_i which is a derivative operator that can give div, grad, or curl, depending on how it's combined with the above. Examples: $(\nabla f)_i = \nabla_i f$, $\nabla \cdot \boldsymbol{A} = \nabla_i A^i$, while

$$(\nabla \times \boldsymbol{A})_i = \epsilon_{ijk} \nabla_j A_k \,. \tag{2}$$

   (a) δ_{ij} is basically the identity matrix. What is $\delta_{ij} A^j$? What is δ_{ii}?

   (b) Similar to what we did in class, show the identity $\nabla \times \nabla f = 0$ for a scalar function f, in index notation.

   (c) Using the identity

$$\epsilon_{ijk} \epsilon_{ilm} = \delta_{jl} \delta_{km} - \delta_{jm} \delta_{kl} \,, \tag{3}$$

   expand $[\nabla \times (\nabla \times \boldsymbol{A})]_i$ in terms of div, grad, and the Laplacian $\nabla^2 = \nabla_i \nabla^i$. You will have to do a bit of index renaming and shuffling around!

   (d) With the "position vector" $\boldsymbol{r}$ that has components $r_1 = x, r_2 = y$, etc., find an index notation expression for

$$\nabla_i r_j \tag{4}$$

2. **Magnetic field of dipole**. In class, we expressed the vector potential for a pure dipole field as

$$\boldsymbol{A}_{\text{dip}}(\boldsymbol{r}) = \frac{\mu_0}{4\pi} \frac{1}{r^2} \boldsymbol{m} \times \hat{\boldsymbol{r}} \,, \tag{5}$$

where $\boldsymbol{m}$ was the magnetic dipole moment. Warning about notation! r^2 looks dangerously close to the second component of the vector $\boldsymbol{r}$. Keep track of what's an exponent and what's an index.

Using index notation, find the magnetic field $\boldsymbol{B}_{\text{dip}}(\boldsymbol{r})$ determined by the vector potential in Eq. (5). Hint: you will need the fact that $\hat{\boldsymbol{r}} = \boldsymbol{r}/r$, the identity from Eq. (3), and the result you found for Eq. (4).

3. Griffiths problem 7.7 (metal bar sliding across rails in a magnetic field).

4. Griffiths problem 7.8 (square loop moved near a current-carrying wire).

5. Griffiths problem 7.22 (self-inductance per unit length of solenoid).