UNIVERSITY OF MISSISSIPPI
Department of Physics and Astronomy
Electromagnetism I (Phys. 401) — Prof. Leo C. Stein — Fall 2019

Problem Set 11 — FINAL

Due: Friday, Dec. 13, 2019, by 5PM

Material: The final covers the material so far (up through and including Griffiths' chapter 6).

Due date: Friday, Dec. 13, 2019 by 5PM to 205 Lewis Hall. If my door is closed, please slide the exam under my door. Late exams will require extenuating circumstances.

Logistics: The exam consists of this page plus two page of questions. Do not look at the problems until you are ready to start it.

Time: The work might expand to eat up as much time as you allot – therefore I highly recommend you restrict yourself to no more than 5 hours cumulative time spent on these problems. You may take as many breaks as you like, not counted against the 5 hours. **You should not be consulting references, working on the problems, or discussing with others during the breaks**.

Resources: The midterm and final are **not collaborative**. All questions must be done on your own, without consulting anyone else. You may consult your own notes (both in-class and notes on this class you or a colleague in the class have made), the textbook by Griffiths, and solution sets on the course website. **You may not consult any other material**, including other textbooks, the web (except for the current Phys. 401 website), material from previous years' Phys. 401 or any other classes, or copies you have made of such material, or any other resources. Calculators and symbolic manipulation programs are not allowed.

1. (a) Is there any charge density ρ that generates the electric field

$$\boldsymbol{E} = \frac{\alpha}{4\pi} \frac{r^2 - 1}{(1 + r^2)^2} \hat{\boldsymbol{r}}\,, \tag{1}$$

where α is some constant? If no, why not? If yes, what is that ρ?

(b) Is there any current density $\boldsymbol{J}$ that can generate the magnetic field

$$\boldsymbol{B} = \alpha \frac{2 + s}{(1 + s)^2} \hat{\boldsymbol{z}}\,, \tag{2}$$

where α is a constant (and as usual $s^2 = x^2 + y^2$ in cylindrical coordinates). If no, why not? If yes, what is that $\boldsymbol{J}$?

2. Suppose there is an infinite straight wire lying along the $+\hat{\boldsymbol{z}}$ axis. We place a charge density λ along this wire, and force those charges to move in the $+\hat{\boldsymbol{z}}$ direction with a steady velocity v.

   (a) What are the electric field $\boldsymbol{E}$ and the magnetic field $\boldsymbol{B}$ created by these charges?

   (b) Now we transport a charge Q from distance b away from the wire to distance a away from the wire. How much work was done on the charge?

   (c) Suppose we give this charge, still at distance a, a velocity $w\hat{\boldsymbol{s}}$ directed away from the wire. What is the total electromagnetic force on this charge?

   (d) Now suppose we take a magnetic dipole $\boldsymbol{m} = m\hat{\boldsymbol{s}}$ pointing away from the wire. We transport this dipole from distance b away from the wire to distance a away from the wire. How much work was done on the dipole?

   (e) What is the torque on this dipole?

   (f) Suppose the dipole has now been rotated (perhaps by the just-computed torque, or some other reason) so that it points in the same direction as the vector $\hat{\boldsymbol{\phi}} + \hat{\boldsymbol{s}}$. What is the force on this dipole?

3. We have placed an insulating shell of radius R centered at the origin. On the surface of this shell, we have distributed charge according to the azimuthally-symmetric surface charge density:

$$\sigma(\theta) = \sigma_0 + \sigma_1 \cos\theta + \sigma_2 \cos^2\theta\,, \tag{3}$$

where $\sigma_{0,1,2}$ are constants.

   (a) Rewrite σ in terms of a series of Legendre polynomials

   (b) What happens to the electric potential $V(r, \theta)$ going across this surface charge? (Give an equation in terms of $\sigma_{0,1,2}$).

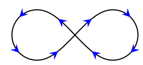
   (c) Find the potential $V(r, \theta)$ inside and outside this shell (again in terms of $\sigma_{0,1,2}$).

4. Show how the Laplace equation can be solved in **cylindrical** coordinates using separation of variables. Show the *ordinary* differential equations (ODE) that result, and state all the conditions on the *separation constants*. You should be able to solve all but one ODE in terms of elementary functions (the last one is solved by a special function we have not yet encountered).

5. (a) Take a sphere of radius R, centered on the origin, that has charge Q distributed uniformly throughout. Cut it into two hemispheres, the "North" ($z > 0$) and "South" ($z < 0$). Discard the Northern hemisphere.

   Go to a very large distance $r \gg R$ and expand the electric field $\boldsymbol{E}$ as a power series in powers of $1/r^k$, with k being positive integers. Find the first two non-zero terms in this series (to avoid any potential issues of conventions, please state your solution as $\boldsymbol{E} = \ldots$).

   (b) Suppose we flow current I through a 'figure 8' shaped wire, with the current going like so:



   The two halves have the same shape, with each loop having radius R, and the central segments crossing at right angles. The wire lies in the $x - y$ plane.

   Go to a very large distance $r \gg R$ and expand the magnetic field $\boldsymbol{B}(r)$ as a power series in powers of $1/r^k$, with k being positive integers. What is the leading term in the series (i.e. lowest k whose coefficient does not vanish) for this current distribution?

6. Let's take a very long cylinder of radius R, with its symmetry axis along the z axis. This cylinder is made of a linear magnetic material with magnetic susceptibility χ_m. The magnetic field inside is

$$\boldsymbol{B} = \frac{\alpha\mu_0}{2}(1 + \chi_m)s^2\,\hat{\boldsymbol{z}}\,, \tag{4}$$

   where α is a constant. Find the following quantities:

   (a) The auxiliary field $\boldsymbol{H}$,

   (b) the magnetization $\boldsymbol{M}$,

   (c) the bound volume current density $\boldsymbol{J}_b$ inside the cylinder,

   (d) the bound surface current density $\boldsymbol{K}_b$ on the surface of the cylinder, and

   (e) the magnetic field $\boldsymbol{B}$ at an infinitesimal distance outside of the cylinder.