UNIVERSITY OF MISSISSIPPI
Department of Physics and Astronomy
Electromagnetism I (Phys. 401) — Prof. Leo C. Stein — Fall 2019

Problem Set 9

Due: Friday, Nov. 22, 2019, by 5PM

As with research, feel free to collaborate and get help from each other! But the solutions you hand in must be your own work. All book problem numbers refer to the third edition of Griffiths, unless otherwise noted. I know we don't all have the same edition, so I also briefly describe the topic of the problem.

1. Griffiths' problem 5.3 (charge to mass ratio of electrons)
2. Griffiths' problem 5.39 (the *Hall effect*)
3. In class we worked out the magnetic field in an idealized coaxial cable where the total return current in the outer conductor balanced with current flowing down the center conductor. Generalize this situation as follows. Suppose we have a coaxial cable running along the $\hat{\boldsymbol{z}}$ axis where the conductor has radius r_1, then a gap, then the outer conductor with inner radius r_2 and outer radius r_3. Suppose the current in the inner conductor is a uniform volume current density $\boldsymbol{J}_{in} = J_{in}\hat{\boldsymbol{z}}$, and the current in the outer conductor is a uniform volume current density $\boldsymbol{J}_{out} = J_{out}\hat{\boldsymbol{z}}$. Find the magnetic field everywhere.
4. **From $\boldsymbol{B}$ to $\boldsymbol{A}$**. Recall that we found the integral form of Ampère's law,
$$\int_{\partial\mathcal{A}} \boldsymbol{B}\cdot d\boldsymbol{l} = \mu_0 \int_{\mathcal{A}} \boldsymbol{J}\cdot d^2\boldsymbol{a} \equiv \mu_0 I_{enc}\,, \tag{1}$$
by starting from the differential version, $\nabla\times\boldsymbol{B} = \mu_0\boldsymbol{J}$, over an area $\mathcal{A}$, and then applying Stokes' theorem.
   (a) Do the same for the definition of the magnetic vector potential, $\nabla\times\boldsymbol{A} = \boldsymbol{B}$, thus arriving at a formula to help find $\boldsymbol{A}$ if you already know $\boldsymbol{B}$.
   (b) Apply this formula to the infinite solenoid of radius R with n windings per unit length, each carrying current I (find $\boldsymbol{A}$ both inside and outside the solenoid).
   (c) Check that the curl and divergence of $\boldsymbol{A}$ from the last step – do they each give the expected result?
   (d) Now find the vector potential $\boldsymbol{A}$ created by an infinite straight wire of radius R, carrying total current I, where the current density is uniform inside the wire (find $\boldsymbol{A}$ both inside and outside the wire).
5. **More identities**.
   (a) Using Stokes' theorem, show that
$$\int_{\mathcal{A}} (\nabla f)\times d^2\boldsymbol{a} = -\oint_{\partial\mathcal{A}} f\,d\boldsymbol{l}\,, \tag{2}$$
   for some scalar function f. (Hint if you're stuck: In Stokes' theorem, consider a vector field that's the product of $\boldsymbol{v} = f\boldsymbol{c}$ for some constant vector field $\boldsymbol{c}$.)
   (b) Using the above identity, show that
$$\oint_{\partial\mathcal{A}} (\boldsymbol{c}\cdot\boldsymbol{r})d\boldsymbol{l} = -\boldsymbol{c}\times\int_{\mathcal{A}} d^2\boldsymbol{a}\,, \tag{3}$$
   for some constant vector $\boldsymbol{c}$. [Hint: consider a scalar function $f = \boldsymbol{c}\cdot\boldsymbol{r}$ in identity (2).]