UNIVERSITY OF MISSISSIPPI
Department of Physics and Astronomy
Electromagnetism I (Phys. 401) — Prof. Leo C. Stein — Fall 2019

Problem Set 8

Due: Monday, Nov. 11, 2019, by 5PM

As with research, feel free to collaborate and get help from each other! But the solutions you hand in must be your own work. All book problem numbers refer to the third edition of Griffiths, unless otherwise noted. I know we don't all have the same edition, so I also briefly describe the topic of the problem.

1. (a) We previously found the electric field of an ideal dipole located at the origin, oriented such that $\boldsymbol{p} = p\hat{\boldsymbol{z}}$. This field was

$$\boldsymbol{E}_{\text{dip}} = \frac{p}{4\pi\epsilon_0 r^3}\left(2\cos\theta\,\hat{\boldsymbol{r}} + \sin\theta\,\hat{\boldsymbol{\theta}}\right) . \tag{1}$$

Show that this can be written as

$$\boldsymbol{E}_{\text{dip}} = \frac{1}{4\pi\epsilon_0 r^3}\left[3(\boldsymbol{p}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}} - \boldsymbol{p}\right] , \tag{2}$$

which is coordinate-independent, and thus valid for any orientation $\boldsymbol{p}$.

(b) Suppose we place an ideal dipole $\boldsymbol{p}$ in an external electric field $\boldsymbol{E}$. Show that the energy of placing this dipole into this external field is

$$U = -\boldsymbol{p}\cdot\boldsymbol{E} . \tag{3}$$

There are multiple acceptable ways to show this.

(c) Now suppose we have two dipoles, $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$, at locations $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$, and let the separation between them be $\boldsymbol{r}_{12} \equiv \boldsymbol{r}_1 - \boldsymbol{r}_2$. These two dipoles push and torque each other. Using the two previous problems, show that the interaction energy between the two dipoles is

$$U = \frac{1}{4\pi\epsilon_0}\frac{1}{r_{12}^3}\left[-3(\hat{\boldsymbol{r}}_{12}\cdot\boldsymbol{p}_1)(\hat{\boldsymbol{r}}_{12}\cdot\boldsymbol{p}_2) + \boldsymbol{p}_1\cdot\boldsymbol{p}_2\right] . \tag{4}$$

2. Suppose we have a ball of radius R that is uniformly polarized with polarization density $\boldsymbol{P}(\boldsymbol{r}') = \boldsymbol{P}$ (i.e. uniform polarization means the polarization density is a constant that's independent of position within the ball). If we directly integrate, the potential due to this source is given by

$$V(\boldsymbol{r}) = \frac{1}{4\pi\epsilon_0}\int_{\mathcal{V}} \frac{\boldsymbol{P}\cdot\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2} d^3\text{Vol}' . \tag{5}$$

As $\boldsymbol{P}$ is constant it can be pulled out of the integral. What is the resulting potential after integrating? Feel free to use a previous result if you remember seeing this integral earlier.

3. Griffiths' [3rd ed.] example 4.7 was a dielectric sphere in a uniform external field. That example was solved as a boundary value problem. The boundary value approach does not have the chicken-and-egg problem of: some field $\boldsymbol{E}_0$ induces a polarization $\boldsymbol{P}_0$, which creates its own field $\boldsymbol{E}_1$, which creates more polarization…

But it's worth seeing what happens if we try to find the fields $\boldsymbol{E}_0, \boldsymbol{E}_1, \boldsymbol{E}_2$, and so on, and sum them all up. Find the pattern for all the terms $\boldsymbol{E}_i$, identify the infinite sum, and compare with Griffiths' result from example 4.7.

4. Recalling that the force on a dipole $\boldsymbol{p}$ is $\boldsymbol{F} = (\boldsymbol{p} \cdot \nabla)\boldsymbol{E}$, the force on a continuum with polarization density $\boldsymbol{P}$ was

$$\boldsymbol{F} = \int (\boldsymbol{P} \cdot \nabla)\boldsymbol{E}_{\text{ext}} d^3\text{Vol} \tag{6}$$

(if the object is in static equilibrium, its own internally-generated electric field do not generate a net force on itself, so only the external electric field counts).

(a) Suppose we take a tiny ball of radius R made of dielectric medium with electric susceptibility χ_e, and place it in an external electric field. What is the force on this ball? (Hint: review Griffiths' example 4.7 [3rd ed.]).

(b) Specialize to the external field being due to a charge q at distance $r \gg R$, and compute the total force on the ball of radius R. Is it attractive or repulsive?

(c) Now instead suppose the external field was due to a pure dipole $\boldsymbol{p}$, again at a distance $r \gg R$. What is the total force on the ball?