**UNIVERSITY OF MISSISSIPPI**
Department of Physics and Astronomy
Electromagnetism I (Phys. 401) — Prof. Leo C. Stein — Fall 2019

## Problem Set 6 — MIDTERM

**Due**: Wednesday, Oct. 16, 2019, by 5PM

**Material:** The midterm covers the material so far (up through and including Griffiths' chapter 3.2).

**Due date:** Wednesday, Oct. 16, 2019 by 5PM to 205 Lewis Hall. If my door is closed, please slide the exam under my door. Late exams will require extenuating circumstances.

**Logistics:** The exam consists of this page plus one page of questions. Do not look at the problems until you are ready to start it.

**Time:** The work might expand to eat up as much time as you allot – therefore I highly recommend you restrict yourself to no more than 5 hours cumulative time spent on these problems. You may take as many breaks as you like, not counted against the 5 hours. **You should not be consulting references, working on the problems, or discussing with others during the breaks**.

**Resources:** The midterm and final are **not collaborative**. All questions must be done on your own, without consulting anyone else. You may consult your own notes (both in-class and notes on this class you or a colleague in the class have made), the textbook by Griffiths, and solution sets on the course website. **You may not consult any other material**, including other textbooks, the web (except for the current Phys. 401 website), material from previous years' Phys. 401 or any other classes, or copies you have made of such material, or any other resources. Calculators and symbolic manipulation programs are not allowed.

1. **Math warm-up**.

   (a) Consider the function $f$ given in cylindrical coordinates,

   $$f(s,\phi,z) \equiv z^2 + s^2 \left( \frac{1}{9}\cos^2\phi + \frac{1}{4}\sin^2\phi \right) . \tag{1}$$

   Describe the surface defined by the level set $f(s,\phi,z) = 1$. Find the unit vector normal to this surface at some arbitrary point.

   (b) Take the vector field $\boldsymbol{v}$ to be given in spherical coordinates as

   $$\boldsymbol{v} = r\sin\phi\,\hat{\boldsymbol{r}} + 2r\sin\theta\cos\phi\,\hat{\boldsymbol{\theta}} + 3r^2\sin\theta\sin\phi\,\hat{\boldsymbol{\phi}} . \tag{2}$$

   Evaluate the flux of $\boldsymbol{v}$ through the following surface $\mathcal{S}$, which is the boundary of the volume where $0 \le r \le 1$, $0 \le \theta \le \pi/3$, and $0 \le \phi \le \pi$.

   (c) Show that the following vector field is conservative:

   $$\boldsymbol{v} = \ln(x^2y + z)\left(2xy\,\hat{\boldsymbol{x}} + x^2\,\hat{\boldsymbol{y}} + \hat{\boldsymbol{z}}\right) . \tag{3}$$

2. **Infinite charged cylinder**. Suppose we are so infinitely powerful that we can construct an infinite cylinder centered on the $\hat{z}$ axis, of diameter $D$. We distribute charge so that the space density is

   $$\rho(s,\phi,z) = \begin{cases} \alpha s & \text{inside,} \\ 0 & \text{outside,} \end{cases} \tag{4}$$

   where $\alpha$ is some positive constant.

   (a) Find the electric field $\boldsymbol{E}$ everywhere (both inside and outside the cylinder).

   (b) Find the electric potential $\boldsymbol{V}$ everywhere (inside and outside).

   (c) Find the amount of work it takes to move a charge $q$ from the center of the cylinder to its edge.

   (d) What force (vector) will the charge $q$ experience if it is just outside the edge of the cylinder?

3. **Charged ball**. Your friend Alice has found that the electric potential due to a specially-prepared charge ball of radius $a$ is

   $$V(\vec{\boldsymbol{r}}) = \begin{cases} \dfrac{\beta}{\epsilon_0}a^2 - \dfrac{\beta}{6\epsilon_0}\dfrac{r^3}{a} & r \le a \\ \dfrac{5\beta}{6\epsilon_0}\dfrac{a^3}{r} & r \ge a\,, \end{cases} \tag{5}$$

   where $\beta$ is a positive constant.

   (a) What is the electric field $\boldsymbol{E}$ everywhere (inside and outside the ball)?

   (b) What is the volume charge density $\rho$ everywhere?

   (c) What is the surface charge density on the surface of the ball at $r = a$?

   (d) Find the total amount of energy stored in the electric field configuration.

4. **Induced charge in a conducting sphere**. One of the cases that we saw could be treated by the method of images was a conducting sphere. Suppose we have a conducting hollow spherical shell of inner radius $R_1$ and thickness $h$ (so the outer radius is $R_2 = R_1 + h$), centered at the origin, with a net zero charge on the conductor. We place a charge $q'$ inside, at coordinates $(0,0,b)$, $0 \le b \le R_1$.

   (a) Where do we imagine an image charge, and what is the value of its charge, in order to find the electric potential $V$ inside (for $0 \le r \le R_1$)? Find $V$ in this region.

   (b) What are the electric field $\boldsymbol{E}$ and potential $V$ in the region $R_1 \le r \le R_2$, and outside, where $R_2 \le r$?

   (c) What is the induced surface charge density $\sigma(\vec{\boldsymbol{r}})$ along the inner surface, $r = R_1$? What about on the outer surface $r = R_2$?

   (d) What is the force (vector) on the charge $q'$ due to this induced surface charge?

   (e) What is the total energy of this configuration?