UNIVERSITY OF MISSISSIPPI
Department of Physics and Astronomy
Electromagnetism I (Phys. 401) — Prof. Leo C. Stein — Fall 2019

Problem Set 4

Due: Friday, Sept. 27, 2019, by 5PM

As with research, feel free to collaborate and get help from each other! But the solutions you hand in must be your own work. All book problem numbers refer to the third edition of Griffiths, unless otherwise noted. I know we don't all have the same edition, so I also briefly describe the topic of the problem.

1. Suppose we create the following charge distribution,

$$\rho(r,\theta,\phi) = \begin{cases} \frac{\kappa}{r^2} & R_1 < r < R_2 \\ 0 & \text{otherwise,} \end{cases} \tag{1}$$

   where κ is some constant. Use Gauss's law to find the $\boldsymbol{E}$ field everywhere in space.

2. What is the $\boldsymbol{E}$ field produced by an infinite slab that stretches in the x, y directions and is restricted to $-h \le z \le +h$ for some positive h, with uniform charge density ρ_0? Find a potential V that corresponds to this $\boldsymbol{E}$.

3. Now we distribute charge along the line segment $y = 0, z = 0$, between $-L \le x \le +L$ for some positive L. But we do not distribute the charge uniformly; instead we apply the charge density per unit length $\lambda(x') = \lambda_0 x'/L$. Find the potential $V(x, y, z)$ created by this charge distribution. You may find the following antiderivatives helpful:

$$\int \frac{du}{\sqrt{u^2+b}} = \log(u + \sqrt{u^2+b})\,, \qquad \int \frac{u\,du}{\sqrt{u^2+b}} = \sqrt{u^2+b}\,. \tag{2}$$

   Then find the $\boldsymbol{E}$ field everywhere.

4. Let's distribute a total charge q throughout the half-sphere that lies in the region $x^2 + y^2 + z^2 \le R$ and $z \le 0$. Find the potential $V(0, 0, z)$ with $z > 0$ along the positive z axis. Now you might find these additional antiderivatives helpful:

$$\int \frac{\sin\theta\,d\theta}{\sqrt{b + c\cos\theta}} = -\frac{2}{c}\sqrt{b + c\cos\theta}\,, \tag{3}$$

$$\int \sqrt{u^2+b}\,du = \frac{u}{2}\sqrt{u^2+b} + \frac{b}{2}\log(u + \sqrt{u^2+b})\,, \tag{4}$$

$$\int u\sqrt{u^2+b}\,du = \frac{1}{3}(u^2+b)^{3/2}\,. \tag{5}$$