UNIVERSITY OF MISSISSIPPI
Department of Physics and Astronomy
Electromagnetism I (Phys. 401) — Prof. Leo C. Stein — Fall 2019

Problem Set 1

Due: Thursday, Sept. 5, 2019, by 5PM

As with research, feel free to collaborate and get help from each other! But the solutions you hand in must be your own work. All book problem numbers refer to the third edition of Griffiths, unless otherwise noted. I know we don't all have the same edition, so I also briefly describe the topic of the problem.

1. Prove the following vector algebra and calculus identities. By prove I mean to show the list of steps with enough detail and justification (e.g. stating "because of antisymmetry of the cross product") so that somebody just learning this topic could follow the derivations, and be convinced of their correctness. Breaking things up into components is a perfectly valid strategy. Boldface symbols are vectors.

$$\boldsymbol{u} \times (\boldsymbol{v} \times \boldsymbol{w}) = \boldsymbol{v}(\boldsymbol{u} \cdot \boldsymbol{w}) - \boldsymbol{w}(\boldsymbol{u} \cdot \boldsymbol{v})\,, \tag{1}$$

$$0 = \boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) + \boldsymbol{b} \times (\boldsymbol{c} \times \boldsymbol{a}) + \boldsymbol{c} \times (\boldsymbol{a} \times \boldsymbol{b})\,, \tag{2}$$

$$\nabla(fg) = (\nabla f)g + f\nabla g\,, \tag{3}$$

$$\nabla \cdot (f\boldsymbol{v}) = (\nabla f) \cdot \boldsymbol{v} + f\nabla \cdot \boldsymbol{v}\,, \tag{4}$$

$$\nabla \cdot (\nabla \times \boldsymbol{v}) = 0\,. \tag{5}$$

2. Let's define the function

$$p(x, y, z) = ax + b^2y^2 - c^2z^2\,, \tag{6}$$

where a, b, c are nonzero real numbers. The set of points with coordinates (x, y, z) that evaluate to $p(x, y, z) = 0$ make a "hyperbolic paraboloid" surface.

   (a) What is the gradient ∇p of this function?
   (b) Find the *unit* normal vector $\hat{\boldsymbol{n}}$ to the surface $p(x, y, z) = 0$.
   (c) Evaluate the unit normal at the point $(1/a, 2/b, \sqrt{5}/c)$.

3. Recall our definition of the vector field $\boldsymbol{r} \equiv x\hat{\boldsymbol{x}} + y\hat{\boldsymbol{y}} + z\hat{\boldsymbol{z}}$. Let's also define a constant vector $\boldsymbol{k}$ with components (k_x, k_y, k_z). Compute the following quantities:

   (a) $\nabla \cdot \boldsymbol{r}$
   (b) $\nabla \times \boldsymbol{r}$
   (c) $\nabla \cdot \hat{\boldsymbol{r}}$
   (d) $\nabla \times \hat{\boldsymbol{r}}$
   (e) $\nabla \times (\boldsymbol{k} \times \boldsymbol{r})$
   (f) $\nabla(\boldsymbol{k} \cdot \boldsymbol{r})$