

Present and future tests of general relativity

Leo C. Stein (TAPIR, Caltech)



EHT16 — Nov. 30, 2016

Why test GR?

$$G_{ab} = 8\pi\hat{T}_{ab}$$

General relativity successful but **incomplete**

- Can't have mix of quantum/classical
- GR not renormalizable
- GR+QM=new physics (e.g. BH information paradox)

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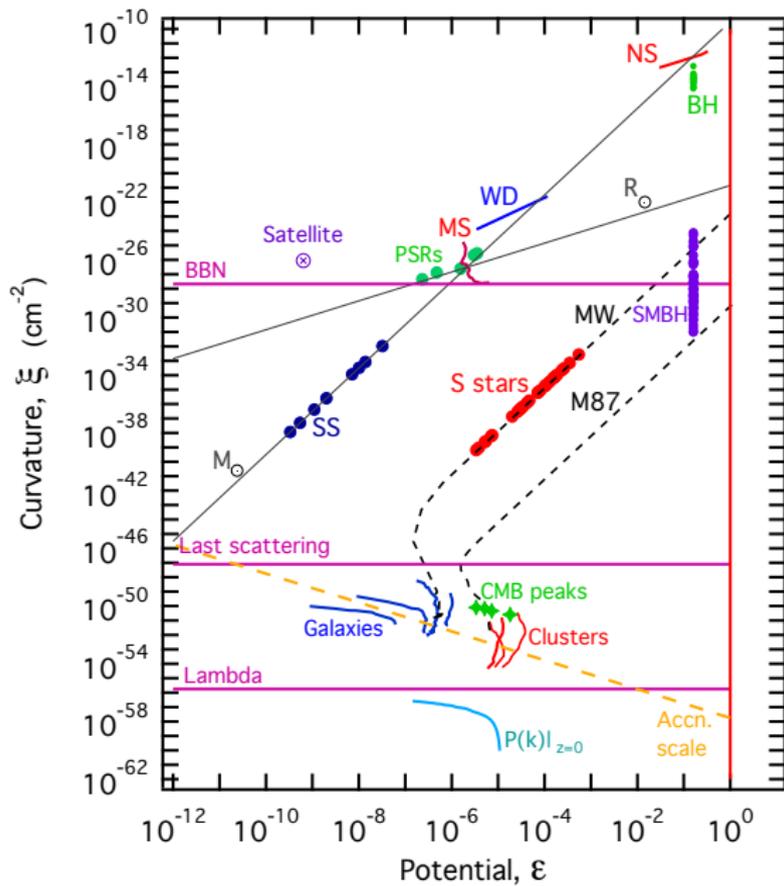
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Empiricism

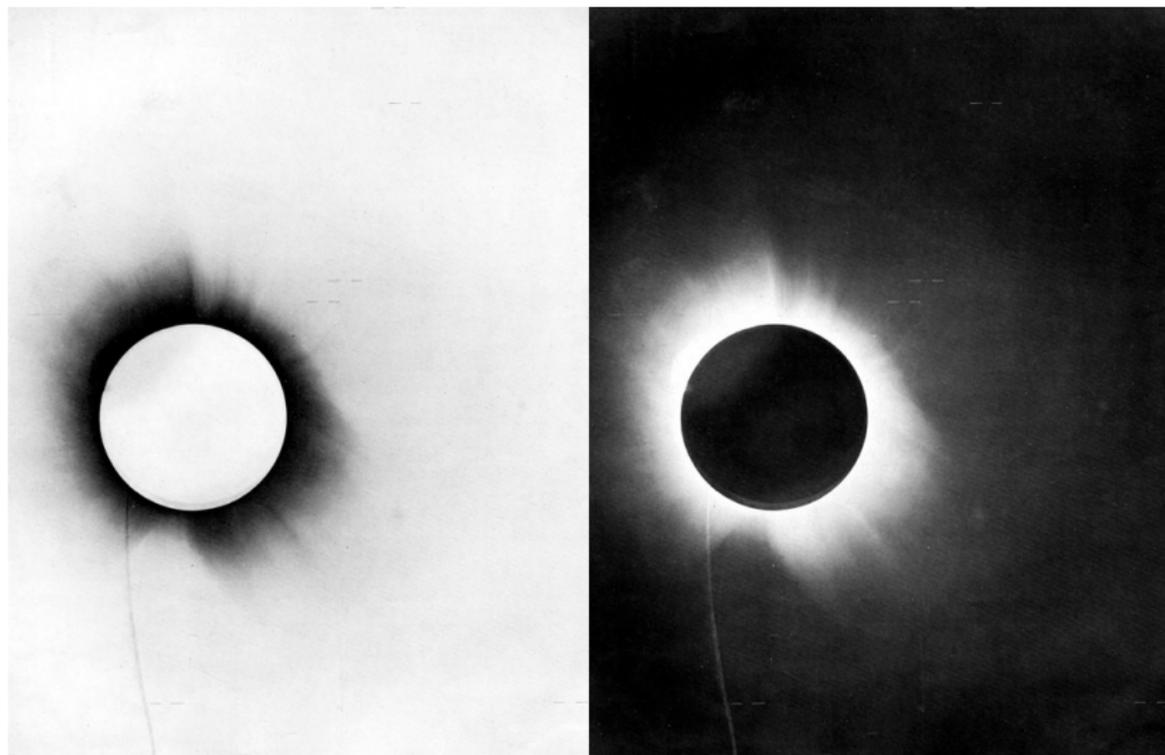
Ultimate test of theory: ask nature



Tests of the past

Eddington 1919

Recommended reading: Kennefick [0709.0685]



Mercury's pericenter precession

- LeVerrier (1859): $526.7''/\text{century}$, discrepant by $43''/\text{century}$.

Venus	Earth	Mars	Jupiter	Saturn	Uranus	Total
280.6	83.6	2.6	152.6	7.2	0.1	526.7

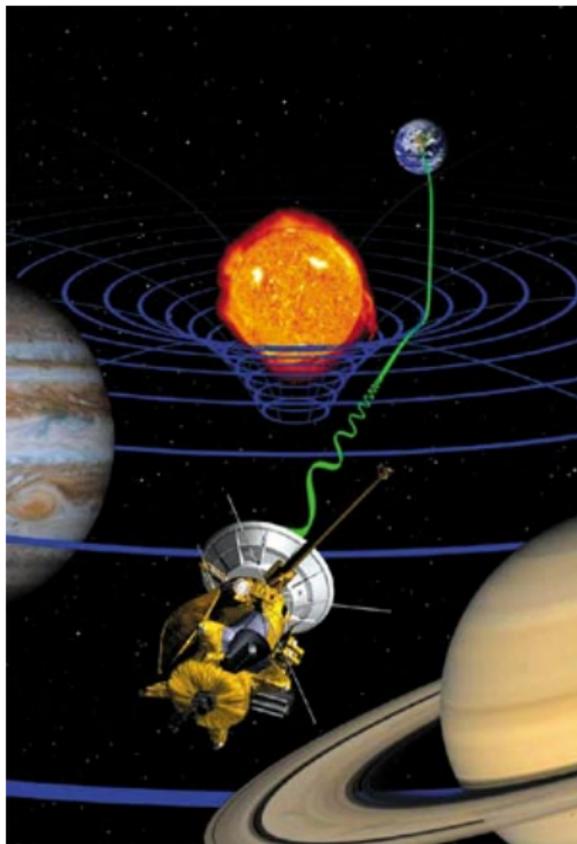
- Einstein to Sommerfeld (Dec. 9, 1915):

“Wie kommt uns da die pedantische Genauigkeit der Astronomie zu Hilfe, über die ich mich im Stillen früher oft lustig mach!”

How helpful to us here is astronomy's pedantic accuracy, which I often used to ridicule secretly!

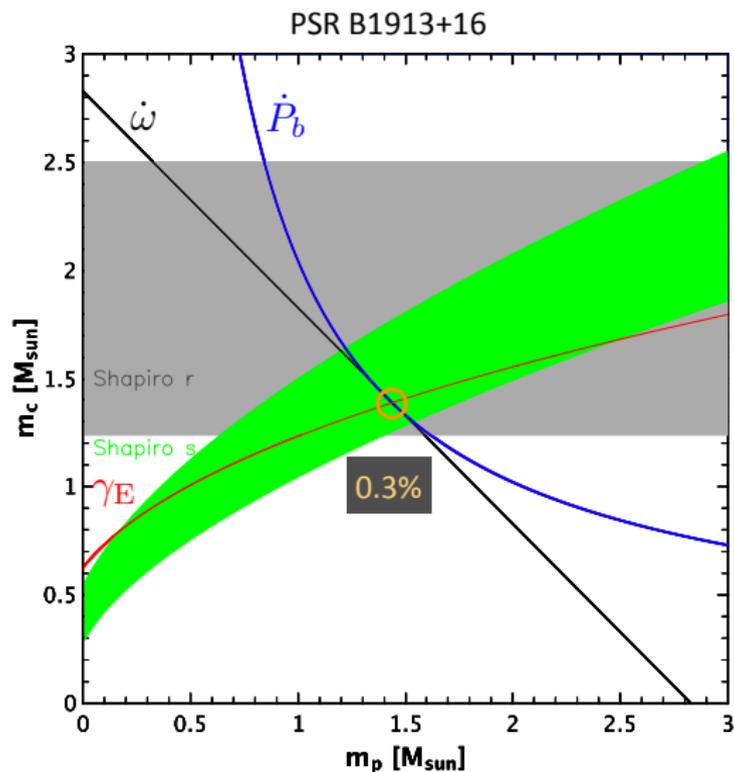


Solar system tests



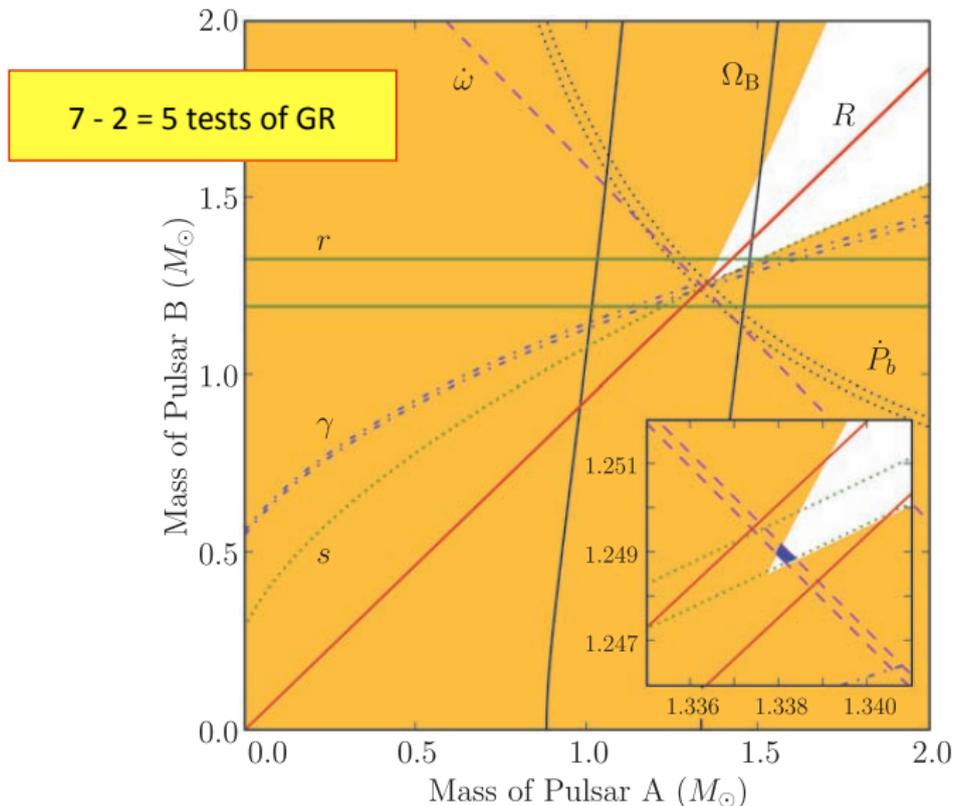
Binary pulsar tests

Keplerian orbits: parameters - observables = 2



Binary pulsar tests

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PPN formalism for metric theories of gravity

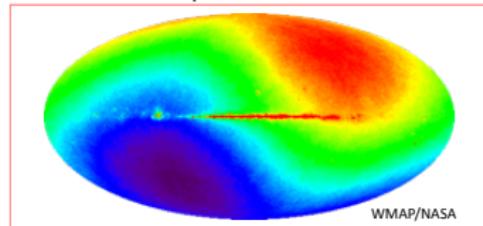
Metric:

$$g_{00} = -1 + 2U - 2\beta U^2 - 2\xi\Phi_W + (2\gamma + 2 + \alpha_3 + \zeta_1 - 2\xi)\Phi_1 + 2(3\gamma - 2\beta + 1 + \zeta_2 + \xi)\Phi_2 \\ + 2(1 + \zeta_3)\Phi_3 + 2(3\gamma + 3\zeta_4 - 2\xi)\Phi_4 - (\zeta_1 - 2\xi)\mathcal{A} - (\alpha_1 - \alpha_2 - \alpha_3)w^2U - \alpha_2 w^i w^j U_{ij} \\ + (2\alpha_3 - \alpha_1)w^i V_i + \mathcal{O}(\epsilon^3),$$

$$g_{0i} = -\frac{1}{2}(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1 - 2\xi)V_i - \frac{1}{2}(1 + \alpha_2 - \zeta_1 + 2\xi)W_i - \frac{1}{2}(\alpha_1 - 2\alpha_2)w^j U_{ij} \\ - \alpha_2 w^j U_{ij} + \mathcal{O}(\epsilon^{5/2}),$$

$$g_{ij} = (1 + 2\gamma U)\delta_{ij} + \mathcal{O}(\epsilon^2).$$

w : motion w.r.t. preferred reference frame



Metric potentials:

$$U = \int \frac{\rho'}{|\mathbf{x} - \mathbf{x}'|} d^3x', \quad (\text{Newtonian potential})$$

$$U_{ij} = \int \frac{\rho'(x-x')_i(x-x')_j}{|\mathbf{x} - \mathbf{x}'|^3} d^3x',$$

$$\Phi_W = \int \frac{\rho'\rho''(\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3} \cdot \left(\frac{\mathbf{x}' - \mathbf{x}''}{|\mathbf{x} - \mathbf{x}''|} - \frac{\mathbf{x} - \mathbf{x}''}{|\mathbf{x}' - \mathbf{x}''|} \right) d^3x' d^3x'',$$

$$\mathcal{A} = \int \frac{\rho'[\mathbf{v}' \cdot (\mathbf{x} - \mathbf{x}')]^2}{|\mathbf{x} - \mathbf{x}'|^3} d^3x',$$

$$\Phi_1 = \int \frac{\rho'v'^2}{|\mathbf{x} - \mathbf{x}'|} d^3x',$$

$$\Phi_2 = \int \frac{\rho'U'}{|\mathbf{x} - \mathbf{x}'|} d^3x',$$

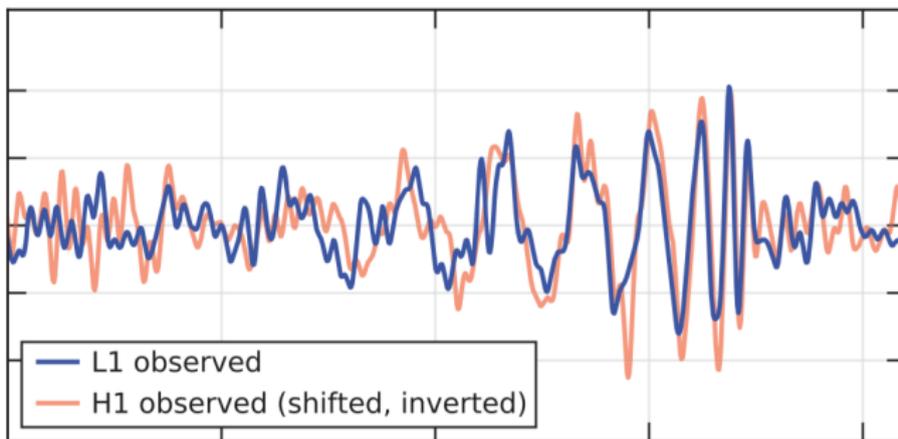
$$\Phi_3 = \int \frac{\rho'\Pi'}{|\mathbf{x} - \mathbf{x}'|} d^3x',$$

$$\Phi_4 = \int \frac{p'}{|\mathbf{x} - \mathbf{x}'|} d^3x',$$

$$V_i = \int \frac{\rho'v'_i}{|\mathbf{x} - \mathbf{x}'|} d^3x',$$

$$W_i = \int \frac{\rho'[\mathbf{v}' \cdot (\mathbf{x} - \mathbf{x}')](x-x')_i}{|\mathbf{x} - \mathbf{x}'|^3} d^3x'.$$

[Will 1993, Will 2014, Living Reviews in Relativity]



PRL **116**, 221101 (2016)

Selected for a [Viewpoint](#) in *Physics*
PHYSICAL REVIEW LETTERS

week ending
3 JUNE 2016



Tests of General Relativity with GW150914

B. P. Abbott *et al.**

(LIGO Scientific and Virgo Collaborations)

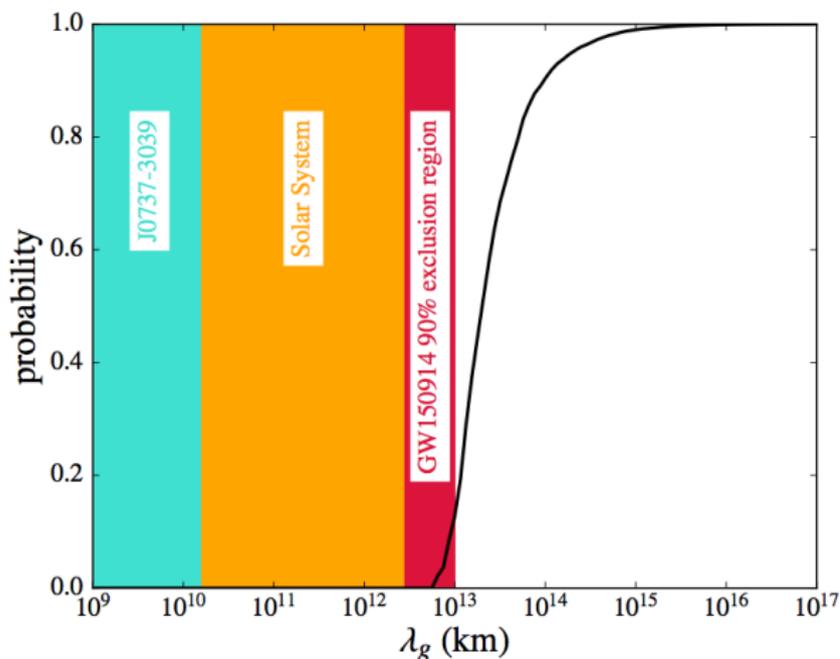
(Received 26 March 2016; revised manuscript received 9 May 2016; published 31 May 2016)

The LIGO detection of GW150914 provides an unprecedented opportunity to study the two-body

LIGO's tests

Two tests I like:

- Any deviation from GR must be below 4% of signal power
- Test of dispersion relation



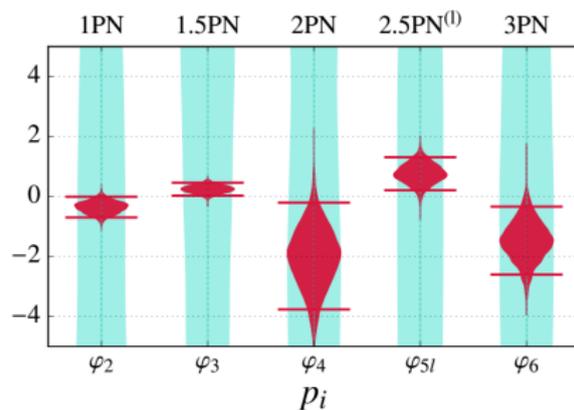
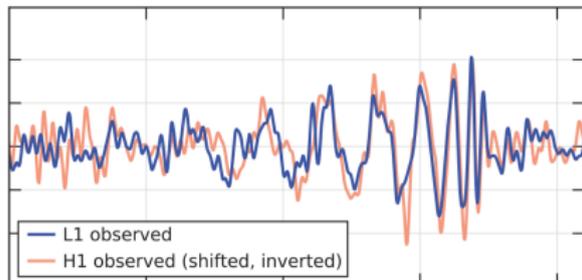
LIGO's tests

One test I do not like:

- Insert power-law corrections to amplitude and phase ($u^3 \equiv \pi \mathcal{M} f$)

$$\tilde{h}(f) = \tilde{h}_{GR}(f) \times (1 + \alpha u^a) \times \exp[i\beta u^b]$$

- Parameters: (α, a, β, b)
- Inspired by **post-Newtonian** calculations in beyond-GR theories



Leo's personal classification of tests

Kinematics vs. Dynamics

Kinematics: study geometry, ignore equations



Dynamics: which equations are being satisfied?

Theory-specific vs. theory-independent

Theory-specific

- Pro: Easy to interpret. Bayesian model comparison
- Con: Lots of work for each theory

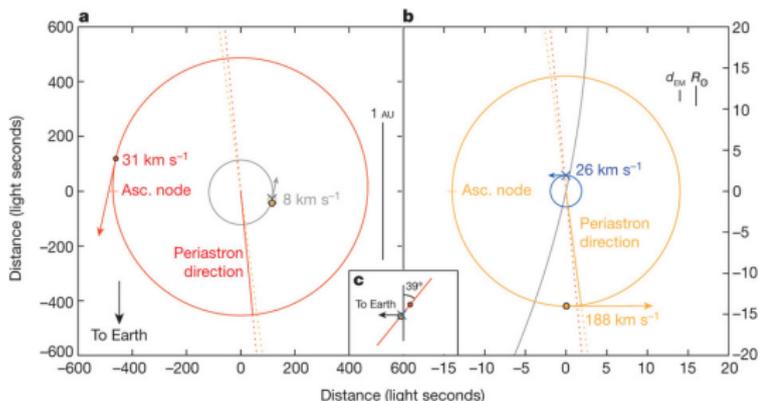
Theory-independent

- Pro: Mapping \implies reuse calculations
- Con: Interpretation unclear. Is parameterization complete?

Tests of today near future

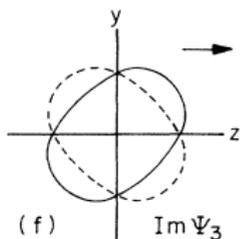
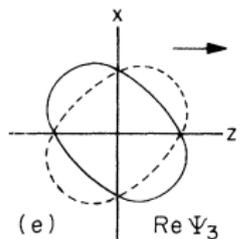
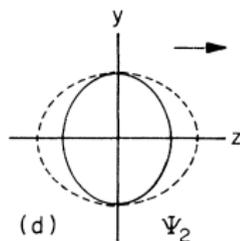
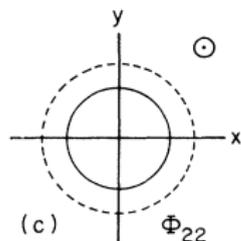
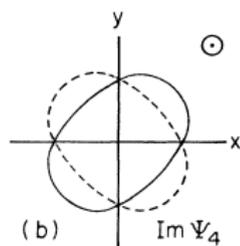
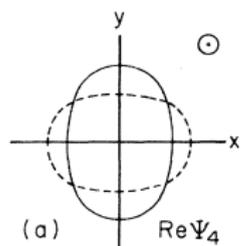
Pulsar timing

- Integrate longer, find more relativistic systems, better technology
- Higher post-Newtonian measurements (I , EOS-dependent)
- Triple system PSRJ0337+1715



- Pulsar around SMBH
- Pulsar timing arrays

Gravitational waves



- More detectors and orientations
- Speed of propagation
- Polarization content

GWs from binary inspirals

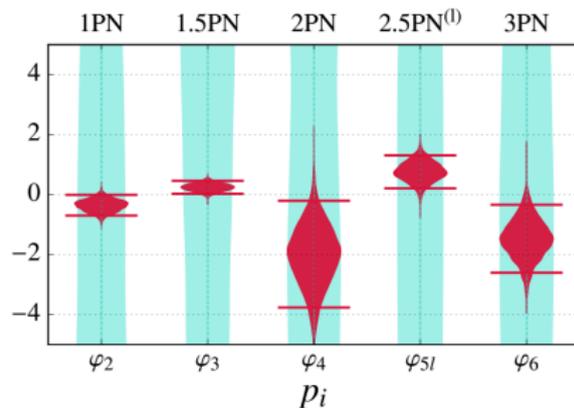
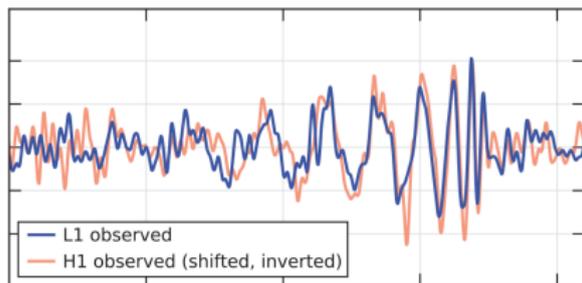
Computed in a few specific theories.

Motivated parameterized post-Einstein framework

- Insert power-law corrections to amplitude and phase ($u^3 \equiv \pi \mathcal{M} f$)

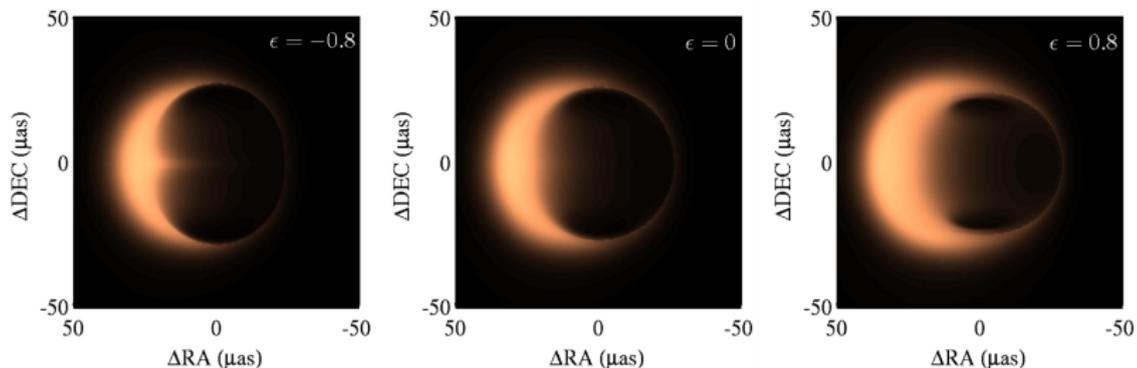
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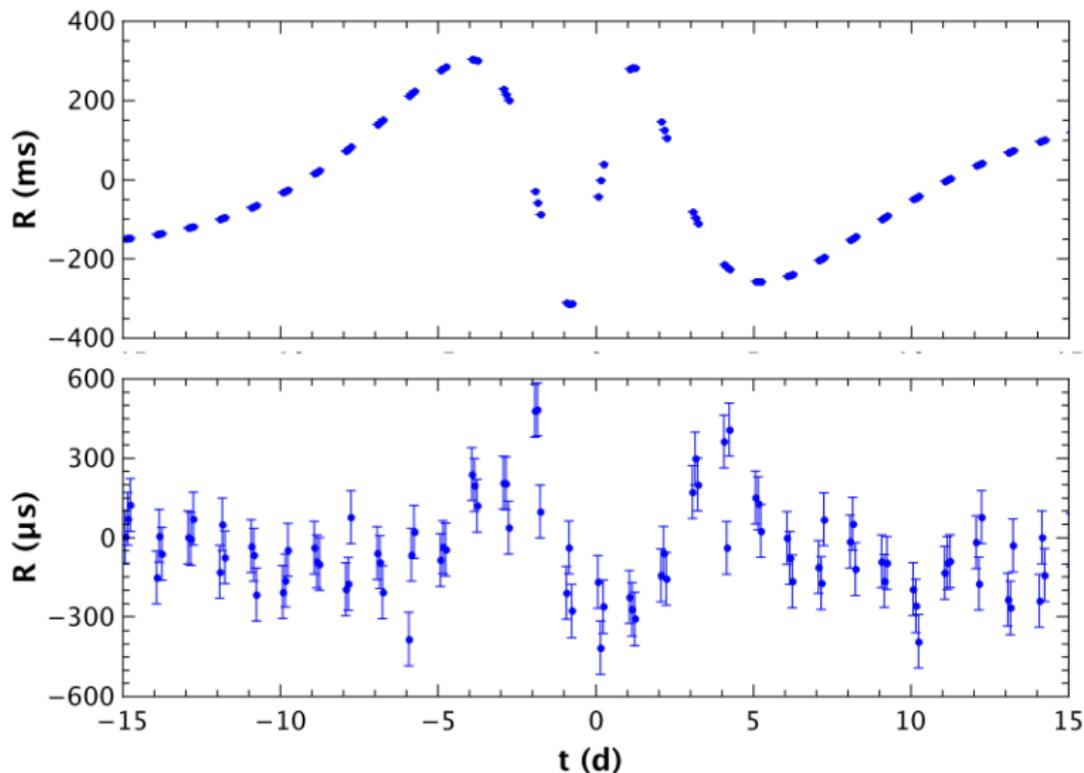
“Bumpy” black holes

- Stationary, axisymmetric spacetimes have **four functions** of two vars.
- Many formalisms to parameterize in countable DOF. Psaltis, Johannsen, Rezzolla, . . .
- Accretion disk modeling, shadow, spectrum. Broderick, Johannsen, Psaltis, . . .



“Bumpy” black holes

- Find pulsar around bumpy SMBH [Psaltis, Wex, Kramer 2015]



Today's shortcomings

- Electromagnetic tests
 - Degeneracy between theory of gravity and plasma prescription, NS EOS
- Theory-specific tests
 - Very few detailed calculations beyond GR
- Theory-independent tests
 - How do parameterizations connect with theories?
 - Are parameterizations sufficient? Well-motivated?
 - Lacking guidance from specific examples

The future

Challenge to the community:

- Investigate degeneracies between matter and gravity.
- Find spacetime solutions in theories beyond GR

Why it's hard

From Lehner+Pretorius 2014:

redshifts of $z \simeq 20$ with a SNR ≥ 10 . For a recent review see Seoane et al. (2013).] Compounding the problem, despite the large number of proposed alternatives or modifications to general relativity (see, for example, Will 1993, 2006), almost none have yet been presented that (a) are consistent with general relativity in the regimes where it is well tested, (b) predict observable deviations in the dynamical strong field relevant to vacuum mergers, and (c) possess a classically well-posed initial value problem to be amenable to numerical solution in the strong field. The notable exceptions are a subset of scalar tensor theories, though these require a time-varying cosmological scalar field for binary black hole systems (Horbatsch & Burgess 2012) or one or more neutron stars in the merger (see Section 5). Thus there is little guidance on what reasonable strong-field deviations one might expect. Proposed solutions to (at least partially) circumvent these problems include the parameterized post-Einsteinian and related frameworks (Yunes & Pretorius 2009, 2010; Will 2014; Will & Flanagan 2015; Flanagan & Will 2007; Will 2008; Will 2011).

- Don't know if other theories have good **initial value problem**
Example: Delsate+ PRD **91**, 024027, dynamical Chern-Simons

A solution

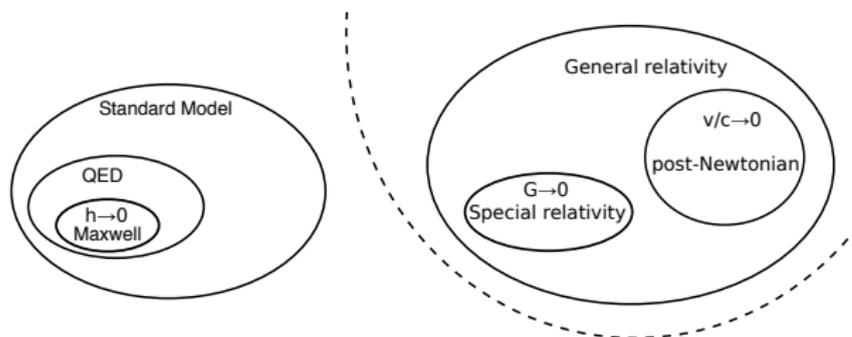
- Treat every theory as an **effective field theory** (EFT)
- Already do this for GR. **Valid** below some scale
- Theory only needs to be **approximate**, approximately well-posed



- Example: weak force below EWSB scale (lose unitarity above)

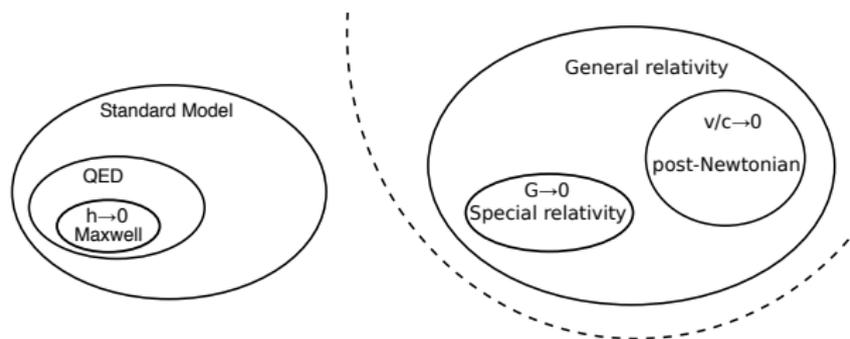
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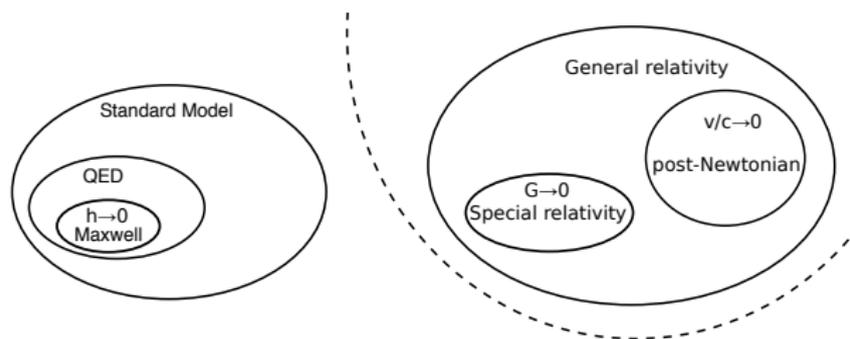
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A solution



- Same should happen in gravity EFT:
lose predictivity (bad initial value problem) above some scale
- Theory valid below cutoff $\Lambda \gg E$. Must recover GR for $\Lambda \rightarrow \infty$.
- Assume **weak coupling**, use **perturbation theory**

A solution

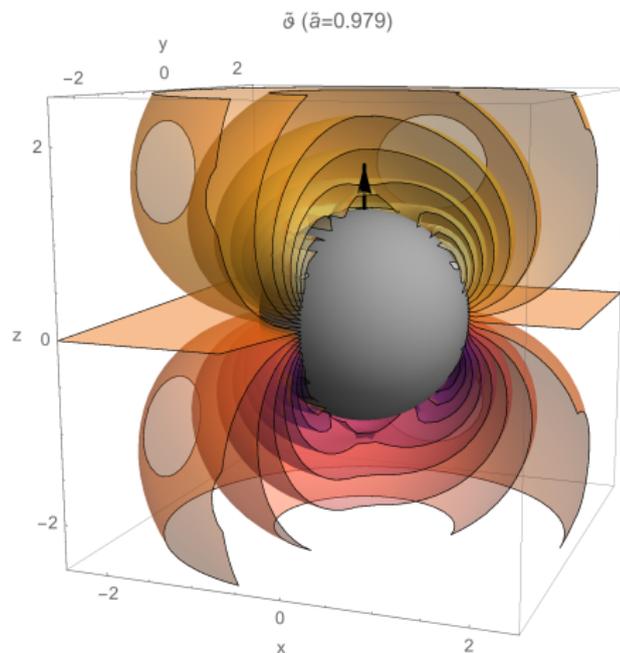


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Example: Dynamical Chern-Simons gravity

Black holes in dCS

- $a = 0$ (Schwarzschild) is exact solution with $\vartheta = 0$
- Rotating BHs have dipole+ scalar hair

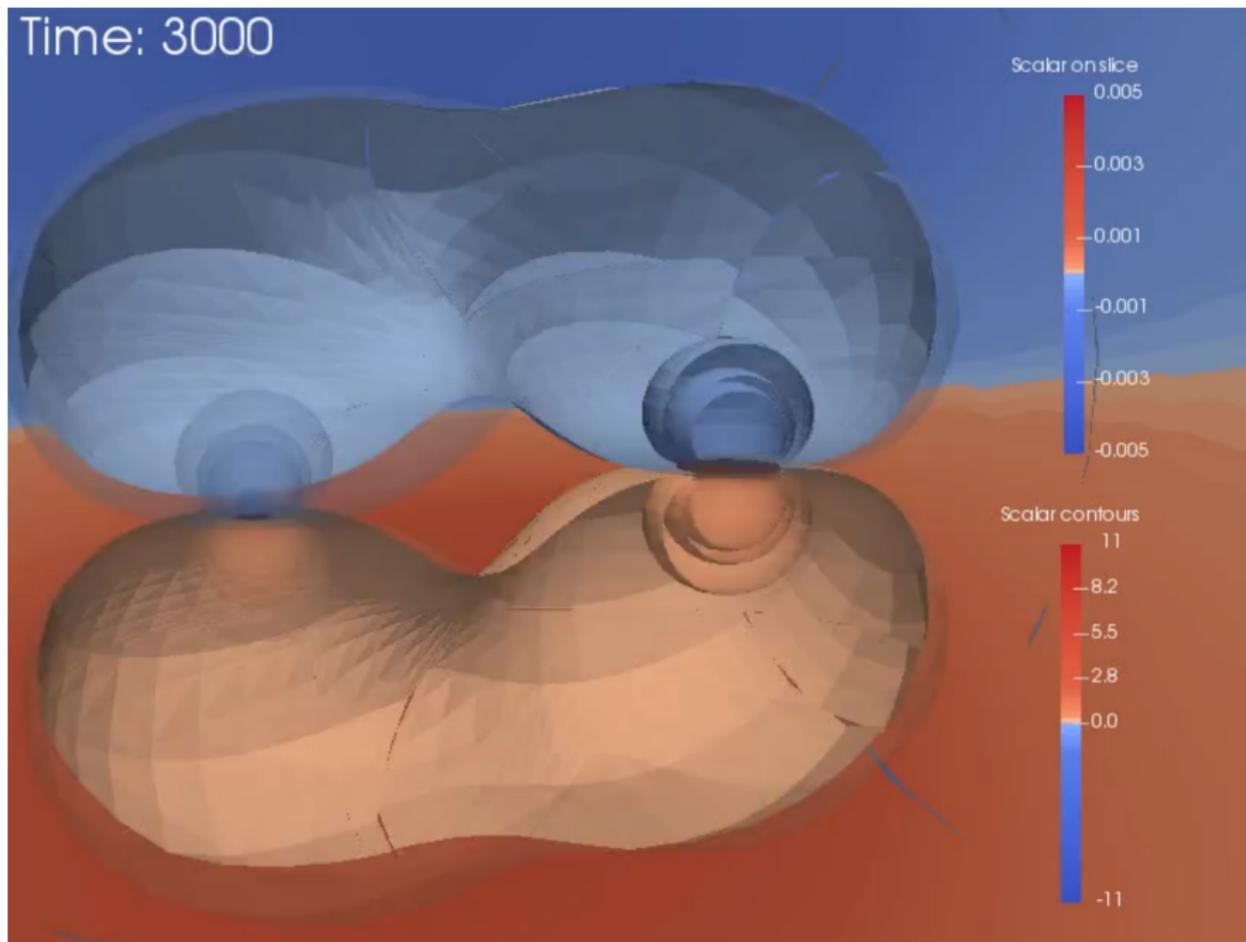


LCS, PRD **90** 044061 (2014) [arXiv:1407.2350]

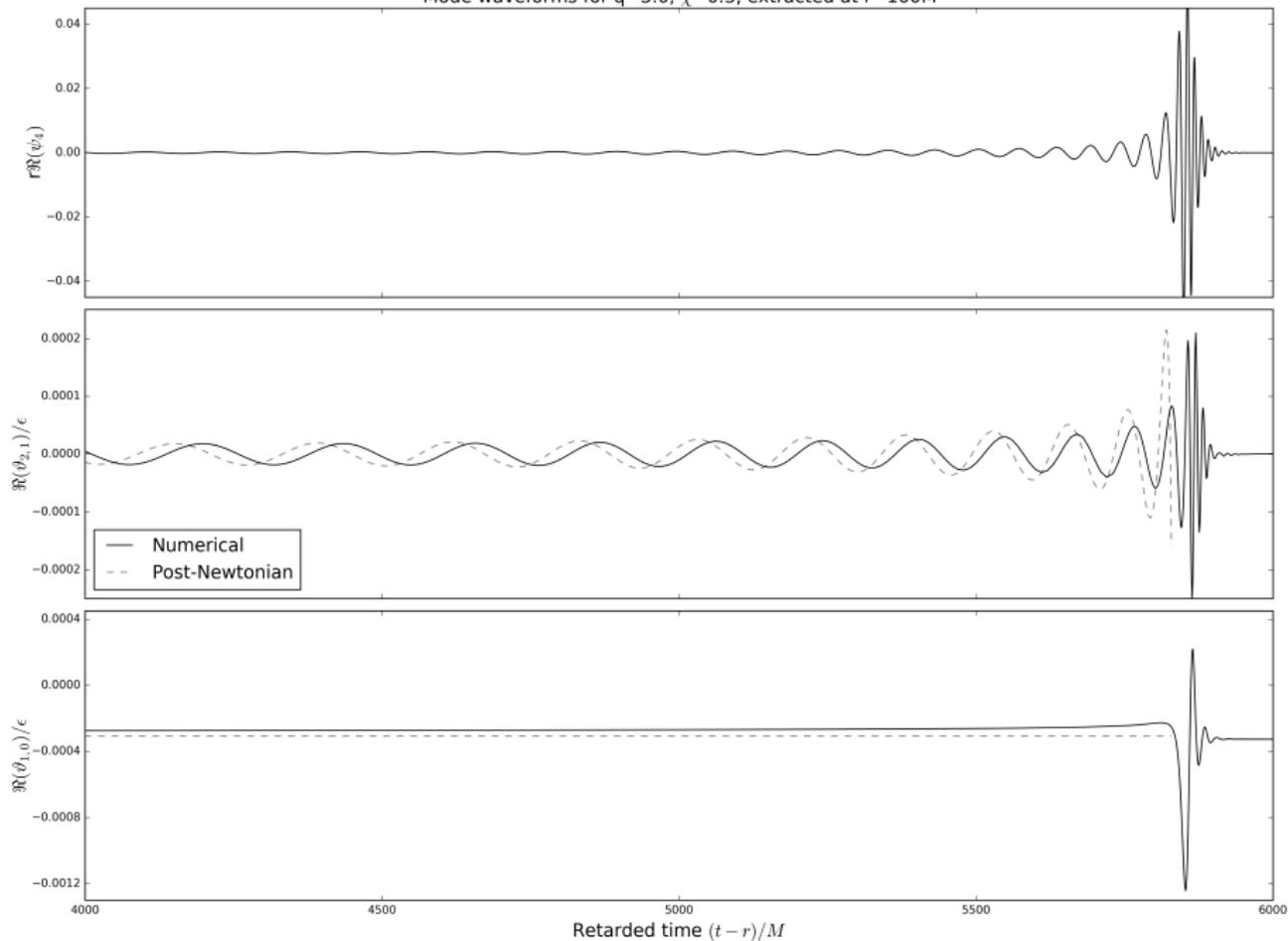
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LCS, PRD **90** 044061 (2014) [arXiv:1407.2350]
- Post-Newtonian of BBH inspiral in
PRD **85** 064022 (2012) [arXiv:1110.5950]
- More updated phenomenology in
CQG **32** 243001 (2015) [arXiv:1501.07274]

Time: 3000



Mode waveforms for $q=3.0$, $\chi=0.3$, extracted at $r=100M$





- General relativity must be incomplete
- New opportunity to test GR in strong-field
- Present tests' shortcomings
 - Almost no theory-specific tests
 - Theory-independent tests need more guidance
- **Challenge:** Find spacetime solutions in theories beyond GR
 - My contribution: First binary black hole mergers in dynamical Chern-Simons gravity