Energy carried by gravitational waves in bimetric gravity

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The TAKEAWAY™

- Two approaches to computing GW energy in bimetric gravity
- Taught xTensor the $e_n(S)$'s, $Y_n(S)^a{}_b$'s, and perturbations
- Found Isaacson-like ESETs
- Found Noether-Wald (pre-)symplectic potential $\boldsymbol{\theta}$ and current $\boldsymbol{\omega}$
- Diagonal on proportional background

Thank you! Questions?

Motivation

- Q: Why energy content?
    - A: Full GW problem (e.g. binary inspiral)
    - Source model
    - Waveform generation
    - Backreaction (easy way): flux balance
- Energy functional is theory-dependent [1012.3144]
- Q: Why Hassan-Rosen bimetric gravity?
    - A: Reasonable, interesting beyond-GR theory
    - No ghosts
    - Only 2nd derivatives
    - *Infrared* correction

Brief history of massive gravity

- Fierz-Pauli: Linear spin-2
  - Ghost-free with tuning $\mathcal{L}_m \supset -\frac{1}{2}m^2(h_{\mu\nu}^2 - h^2)$
  - New length scale: $\lambda_C \sim m^{-1}$
- Problems: vDVZ discontinuity, Boulware-Deser ghost
- Ghost-free to all orders: dRGT
- Ghost-free and diff-invariant: Hassan-Rosen

$$S = m_{pl}^2 \int \left[\sqrt{|g|}R(g) + \alpha^2 \sqrt{|f|}R(f) - 2m^2 \sqrt{|g|} \sum_{n=0}^{4} \beta_n e_n(S) \right]$$

where $S^2 \equiv g^{-1}f$, and $e_n(S)$: elem. symm. poly., e.g.

$$e_2(S) = \frac{1}{6}(\llbracket S \rrbracket^3 - 3\llbracket S \rrbracket \llbracket S^2 \rrbracket + 2\llbracket S^3 \rrbracket)$$

Isaacson approach

- Perturbation theory, all fields $g = g^{(0)} + \epsilon\,\delta g$
- In GR: second order equations,

$$G_{\mu\nu}[g^{(0)}] = -\left\langle G^{(2)}[\delta g, \delta g]\right\rangle \equiv 8\pi T^{\text{eff}}_{\mu\nu}$$

- Need to find 2$^{\text{nd}}$ order perturbations of bigravity EOMs,

$$\mathcal{G}_{\mu\nu} + m^2 V_{\mu\nu} = 0\,,$$
$$\tilde{\mathcal{G}}_{\mu\nu} + \frac{m^2}{\alpha^2}\tilde{V}_{\mu\nu} = 0\,,$$

where

$$V \supset \sum_n \beta_n Y_n(S)\,, \qquad Y_n(S) = \frac{\partial e_{n+1}(S)}{\partial S^T}$$

Isaacson approach

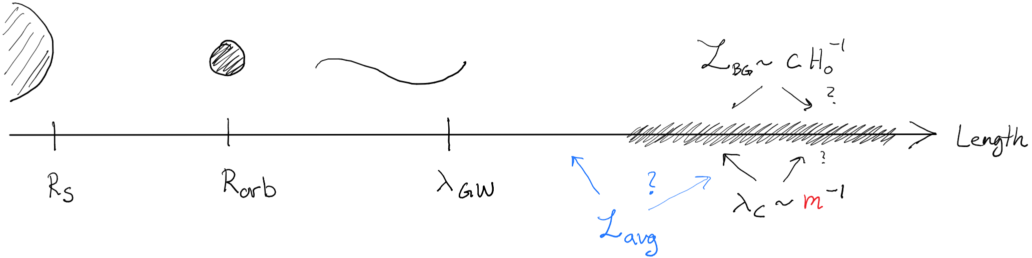
- Second order perturbations laborious but mechanical
- Taught xAct/xTensor suite how to calculate with:
  - Index-free matrix products, powers, traces
  - Elementary symmetric polynomials $e_n(S)$
  - Their derivatives $Y_n(S)^{\mu}{}_{\nu}$
  - Perturbations of these and $\delta S \supset \delta g', \delta f'$
- For example:

$$\delta^2(gY_2(S)) = 2\delta g\, S\, \delta S + 2g\, \delta S\, \delta S + (\text{8 more})$$
$$\delta^2(Y_2(S)) = 2g^{-1}\delta g'\, g^{-1}\, f\, [\![\delta f'\, f^{-1}]\!] + (\text{14 more})$$

- Not illuminating, but have the tensor to be averaged

Issues with Isaacson approach

- What length scale $\mathcal{L}_{\text{avg}}$ for averaging?



- According to which metric?
- Lose effect of m?
- *Two* ESETs (one in each EOM). What is total energy flux?

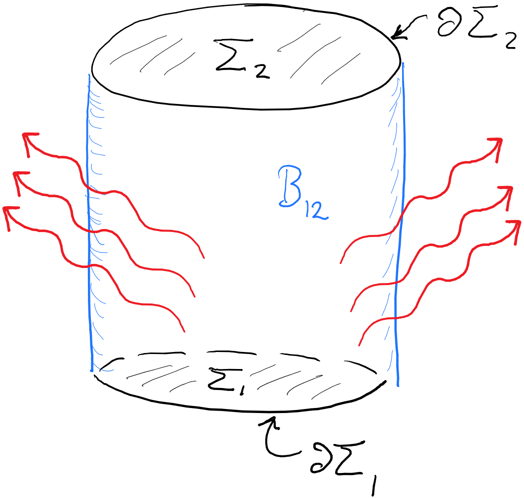
Noether-Wald approach

- Following Wald and Zoupas (2000)

$$
\begin{gathered}
\boldsymbol{L} \\
\downarrow \\
\delta \boldsymbol{L} = \boldsymbol{E}\delta\phi + d\boldsymbol{\theta} \\
\downarrow \\
\boldsymbol{\omega}(\phi, \delta_1\phi, \delta_2\phi) = \delta_1\boldsymbol{\theta}(\phi, \delta_2\phi) - (1 \leftrightarrow 2) \\
\downarrow \\
\delta H_\xi\Big|_{\delta\Sigma_2} - \delta H_\xi\Big|_{\delta\Sigma_1} = -\int_{\mathcal{B}_{12}} \boldsymbol{\omega}(\phi, \delta\phi, \mathcal{L}_\xi\phi) \\
\downarrow \\
H_\xi\Big|_{\delta\Sigma_2} - H_\xi\Big|_{\delta\Sigma_1} = -\int_{\mathcal{B}_{12}} \boldsymbol{\Theta}(\phi, \mathcal{L}_\xi\phi)
\end{gathered}
$$

- Gives flux of Noether current associated to generator ξ
- Pre-symplectic potential $\boldsymbol{\theta}$ and current $\boldsymbol{\omega}$ arise from *derivative terms*
- Copy GR results

Noether-Wald approach



Noether-Wald approach

- Ex.: GR results

$$\omega^{\text{GR}}_{abc} = \frac{1}{16\pi} w^d \epsilon_{dabc} \,,$$
$$w^a = P^{abcdef} [\gamma_{2bc} \nabla_d \gamma_{1ef} - (1 \leftrightarrow 2)] \,,$$
$$P^{abcdef} = g^{ae} g^{bf} g^{cd} + (\text{5 more terms})$$

- Recycle:

$$\omega^{\text{HR}} = m^2_{pl} [\underset{g}{w} \cdot \underset{g}{\epsilon} + \alpha^2 \underset{f}{w} \cdot \underset{f}{\epsilon}] \,,$$
$$\underset{g}{w} = \underset{g}{P}^{\cdots} [\delta g_2 \underset{g}{\nabla} \delta g_1 - (1 \leftrightarrow 2)]$$

Issues with Noether-Wald approach

- Only understand in asymptotically flat
- Undermine cosmological motivation for massive gravity?
- Very difficult in general background
- So far worked on proportional background $f_{\mu\nu} = c^2 g_{\mu\nu}$

With proportional background

- Using results from Bernard+ (2016)
- Mass eigenstates

$$\delta G_{\mu\nu} = \delta g_{\mu\nu} + \alpha^2 \delta f_{\mu\nu}\,, \qquad \delta M_{\mu\nu} = \frac{1}{2c}\left(\delta f_{\mu\nu} - c^2 \delta g\right)$$

- Plug into ω, get diagonal result:

$$\omega^{\mathsf{HR}}_{abc} = m_{pl}^2 \frac{1}{1 + c^2\alpha^2} \underset{g}{\epsilon} \underset{g}{P}^{\cdots} \left[\delta G_2 \underset{g}{\nabla} \delta G_1 + 4\alpha^2 \delta M_2 \underset{g}{\nabla} \delta M_1 - (1 \leftrightarrow 2)\right]$$

Future work

- Understand asymptotics
- Relax proportional background?
- Evaluate with binary GW solution

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